Modeling the dependence between a Poisson process and a continuous semimartingale

Application to electricity spot prices and wind production modeling

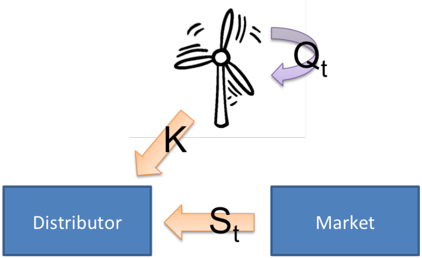
Thomas Deschatre[1,2]

[1]CEREMADE, University of Paris Dauphine and PSL

[2]EDF Lab and FiME Lab

March 2018

Motivation



$$\int Q_t (S_t - K)\, dt$$

- Dependence between S and Q can be modeled by a correlation
- but it does not take into account dependence in extreme events.
- However, spikes can be caused by high renewable production.
- This dependence can have an impact on the distribution of the incomes.
- General problem : modeling and estimating the dependency between extreme events and exogenous variable.

Outline

1. Dependence between a jump process and a continuous time exogenous factor

2. A joint model for the German electricity spot price and wind penetration index (in collaboration with Almut Veraart)

Outline

1. Dependence between a jump process and a continuous time exogenous factor
2. A joint model for the German electricity spot price and wind penetration index (in collaboration with Almut Veraart)

Statistical setting

- We observe an Itô continuous semimartingale on $[0, T]$

$$X_t = \int_0^t \mu_s ds + \int_0^t \sigma_s dW_s.$$

- We observe a Poisson process $(N_t)_{0 \leq t \leq T}$ with stochastic intensity $(\lambda_t)_{0 \leq t \leq T}$.
- We assume that $\lambda_t = nq(X_t)$, $t \in [0, T]$; n corresponds to the asymptotic.

Objectives

- Estimate q on a given interval I and evaluate its performance relatively to the $L_2(I)$ norm,
- Develop a test in order to determine if q belongs to some parametric family on I.

Local time of X

Theorem (from Barlow and Yor (1982); Revuz and Yor (2013); Yen and Yor (2013))

Let us assume that :

(i) $\sigma_s \geq \underline{\sigma} > 0$ *a.s. and* $\mathbb{E}\left(\int_0^T |\mu_s|ds + \sup_{t\in[0,T]} |\int_0^t \sigma_u dW_u|\right) < \infty$ *or*

(ii) $X_t = t$.

Then, there exists a function defined on $\mathbb{R} \times [0, T]$ *and denoted* $(x, t) \mapsto l_t^x$ *verifying*

(i) *an occupation time formula* $\int_0^t f(X_s)\, ds = \int_{\mathbb{R}} f(x)\, l_t^x dx$ *for any measurable function* f *on* $\Omega \times \mathbb{R}$,

(ii) $\mathbb{E}\left(\sup_{x\in\mathbb{R}} l_T^x\right) < \infty$ *and*

(iii) $x \mapsto l_T^x$ *is continuous on* $\mathbb{R}$ *under (i) and has one point of discontinuity under (ii).*

l_T^x measures the time spent by X around the point x before time T
$\Longrightarrow$ must be large enough to estimate q at point x.

We work on $D(I, \nu) = \{\omega \in \Omega,\ \inf_{x\in I} l_T^x(\omega) \geq \frac{\nu T}{|I|}\}$, $\nu \in (0, 1]$.

Local polynomial estimator

Let K a positive kernel, $K_h(\cdot) = \frac{1}{h} K\left(\frac{\cdot}{h}\right)$, $U(x) = \left(1, x, \frac{x^2}{2!}, .., \frac{x^m}{m!}\right)^T$ and $h \in \mathcal{H} \subset (0, \infty)$.

We define the local polynomial estimator $\hat{q}_h = U^T(0)\,\hat{\theta}_h$ with

$$\hat{\theta}_h = \underset{\theta \in \mathbb{R}^{m+1}}{argmin} \; -\frac{2}{n}\theta^T \int_0^T U\left(\frac{X_s - x}{h}\right) K_h(X_s - x)\,\mathbf{1}_{X_s \in I} dN_s$$

$$+ \theta^T \int_0^T U\left(\frac{X_s - x}{h}\right) U^T\left(\frac{X_s - x}{h}\right) K_h(X_s - x)\,\mathbf{1}_{X_s \in I} ds\;\theta.$$

Theorem

On $D(I, \nu)$, if there exists $\Delta > 0$ such that K is bounded from below by a constant > 0 on $[-\Delta, \Delta]$, $\hat{q}_h$ is well defined on I for $h \leq \frac{2}{3}|I|\Delta$.

Kernel estimator case (m = 0) :

$$\hat{q}_h(x) = \frac{1}{n} \frac{\int_0^T K_h(X_s - x)\,\mathbf{1}_{X_s \in I} dN_s}{\int_0^T K_h(X_s - x)\,\mathbf{1}_{X_s \in I} ds}.$$

Bandwidth selection

- We want to minimize $\mathbb{E}\left(\|q-\hat{q}_h\|_I^2 | D(I,\nu)\right)$ w.r.t. h.
- Use of the method of Lacour et al. (2016) in the context of i.i.d observations and density estimation.
- When $h_{\min} = \min \mathcal{H} \to 0$, the bias $\|q_h - q\|_I^2 \approx \|q_h - q_{h_{min}}\|_I^2$.
  $\Longrightarrow$ An unbiased estimator of the bias is

$$\hat{B}_h = ||\hat{q}_h - \hat{q}_{h_{min}}||_I^2 + \text{ bias correction term.}$$

- An unbiased estimator of the variance (kernel estimator case) is

$$\hat{V}_h = \frac{1}{n^2}\int_0^T \int_I \frac{K_h(X_s - x)\mathbf{1}_{X_s\in I}}{\left(\int_0^T K_h(X_u - x)\mathbf{1}_{X_u\in I} du\right)^2} dx dN_s.$$

- We choose the bandwidth

$$\hat{h} = \underset{h\in\mathcal{H}}{argmin} \left(\hat{B}_h + \kappa \hat{V}_h\right), \ \kappa > 0.$$

Oracle inequality

Use of concentration inequalities from Reynaud-Bouret (2014) and Houdré and Reynaud-Bouret (2003).

Theorem

Let us suppose that $\min\ \mathcal{H} \geq \frac{||K||_\infty ||K||_1 |I|}{n}$ *and* $\max\ \mathcal{H} \leq \frac{2}{3}|I|\Delta$.

$$\mathbb{E}\left(||\hat{q}_{\hat{h}} - q||_I^2 | D(I,\nu)\right) \leq \left(\kappa \vee \frac{1}{\kappa} + \frac{\tilde{C}_1}{\log(n)}\right) \min_{h \in \mathcal{H}} \mathbb{E}\left(||\hat{q}_h - q||_I^2 | D(I,\nu)\right)$$

$$+ \tilde{C}_2(\kappa) \log(n) \mathbb{E}\left(||q_{h_{\min}} - q||_I^2 | D(I,\nu)\right)$$

$$+ \frac{\tilde{C}_3(K,\kappa)|I|}{\nu^2 T^2}\left(\frac{\left(1 + ||q||_{I,\infty} \mathbb{E}\left(||I_T||_{I,\infty} | D(I,\nu)\right)|I|\right) \log(n \vee |\mathcal{H}|)^5}{n} + \frac{\log(n \vee |\mathcal{H}|)^6}{n}\right)$$

$$+ \tilde{C}_4 \frac{||q||_I^2}{n^4} + \frac{\tilde{C}_5(K)|I|}{\nu^2 T^2} \frac{\sqrt{\sum_{i=1}^4 \left(||q||_{\infty,I} T\right)^i}}{n}.$$

Minimax optimality

- Let $\Sigma(\beta, I)$ is the Hölder class of order β on I,
- $\Lambda_{\rho,\beta} = \{f : I \to \mathbb{R} : \ f(x) \geq \rho, \ ||f||_{I,\infty} < \infty\} \cap \Sigma(\beta, L, I)$ and
- $\mathbf{R}(\tilde{q}_n, \Lambda_{\rho,\beta}, \varphi_n) = \sup_{q \in \Lambda_{\rho,\beta}} \mathbb{E}\left(\varphi_n^{-2} \int_I (\tilde{q}_n - q(x))^2 \, dx | D(I, \nu)\right)$ the minimax risk.

Theorem (Minimax optimality)

If $\mathcal{H} = \{h > 0 | h \geq \frac{||K||_\infty ||K||_1 |I|}{n}, \ h \leq \frac{2}{3}|I|\Delta$ *and* $|I| h^{-1} \in \mathbb{N}\}$*,*

$$\liminf_{n\to\infty} \inf_{\tilde{q}_n} \mathbf{R}\left(\tilde{q}_n, \Lambda_{\rho,\beta}, n^{\frac{-\beta}{2\beta+1}}\right) > 0.$$

$$\limsup_{n\to\infty} \mathbf{R}\left(\hat{q}_{\hat{h}}, \Lambda_{\rho,\beta}, n^{\frac{-\beta}{2\beta+1}}\right) < \infty \text{ if } m \geq \lfloor\beta\rfloor,$$

$$\limsup_{n\to\infty} \mathbf{R}\left(\hat{q}_{\hat{h}}, \Lambda_{\rho,\beta}, n^{\frac{-m}{2m+1}}\right) < \infty \text{ if } m \leq \lfloor\beta\rfloor,$$

Parametric versus non parametric test (kernel estimator case)

$$\text{Test} \left\{ \begin{array}{l} H_0 : \ \exists \theta_0 \in \Theta \subset \mathbb{R}^d, \ q(\cdot) = g_\theta(\cdot) \ \text{against} \\ H_1 : \ q \neq g_\theta \ \forall \theta \in \Theta \end{array} \right. .$$

$$\text{Let } M_n(\theta) = ||\hat{q}_{\hat{h}}(\cdot) - \int_0^T \frac{K_{h_n}(X_s - \cdot)}{\int_0^T K_{h_n}(X_u - \cdot) \mathbf{1}_{X_u \in I} du} \mathbf{1}_{X_s \in I} g_\theta(X_s) \, ds||_I^2 + \text{bias correction term},$$

$$\hat{\theta}_n = \underset{\theta \in \Theta}{\text{arginf}} \, M_n(\theta).$$

Under H_0, $\hat{\theta}_n \overset{p}{\to} \theta_0$ at the speed $\sqrt{n}$ and we

$$\text{reject } H_0 \text{ if } n\sqrt{\hat{h}} M_n\left(\hat{\theta}_n\right) \geq \sqrt{\hat{V}_n} \Phi^{-1}(1-\alpha)$$

with $\hat{V}_n = C(K) \int_I \left(\frac{g_{\hat{\theta}_n}(y)}{\int_0^T K_{\hat{h}}(y - X_s) \mathbf{1}_{X_s \in I} ds} \right)^2 dy.$

Resume

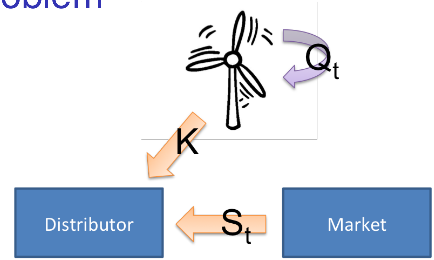
We propose :

- a model taking into account the dependence of the extreme events with an exogenous variable,
- a local polynomial estimator of the intensity of jumps as a function of the exogenous covariate which is optimal in the minimax sense over the Hölder classes,
- a way to choose optimally the bandwidth of the estimator in a non asymptotic framework with an oracle inequality,
- a test to determine if the intensity as a function of the exogenous covariate belongs to a parametric family, which can be useful for operational purposes.

Outline

1. Dependence between a jump process and a continuous time exogenous factor
2. A joint model for the German electricity spot price and wind penetration index (in collaboration with Almut Veraart)

Recall of the problem



$$\int Q_t (S_t - K)\, dt$$

- Spikes of S can be caused by high renewable production Q.
- Need to model this dependence to capture all the risks.

Use of Wind penetration index = $\dfrac{\text{wind energy production}}{\text{electricity load}}$

Objectives

- Propose a joint model of electricity spot price and wind penetration index with dependence in the extremes.

Data

The data sets are considered from the 01/01/2012 to the 31/12/2016 and are the following ones :

- The German and Austrian hourly electricity spot prices (from the day-ahead auction),
- the German and Austrian hourly load data,
- the German and Austrian hourly wind energy production data. The data has been aggregated over the four German transmission system operators 50 Hertz Transmission, Amprion, Tennet TSO and EnBW Transportnetze and the Austrian transmission system operator APG.

Model

$$S_t = \Gamma_{1,t} + X_{1,t} + \sum_{i=1}^{N_t} K_i e^{-\beta(t-T_i)}$$

$$WP_t = expit\left(\Gamma_{2,t} + X_{2,t}\right)$$

observed on a regular grid with steps Δ_n where

- $expit\,(x) = \frac{1}{1+e^{-x}}$,
- Γ_1 and Γ_2 are seasonality functions,
- X_1 and X_2 are centered continuous stochastic processes,
- N_t is a Poisson process with stochastic intensity λ_t, K_i and T_i are the sizes and the times of the jumps.
- β is the speed of the mean reversion of the spikes and is large.

Two structures of dependence :

- a constant correlation between X_1 and X_2,
- the intensity $\lambda_t = q\,(WP_t)$.

Jump detection

We consider the method of Deschatre et al. (2018) to detect spikes when β is large. There is a spike in $((i-1)\Delta_n, i\Delta_n]$ iif

- $\Delta_i^n S = S_{i\Delta_n} - S_{(i-1)\Delta_n} > 5\hat{\sigma}\Delta_n^{0.49}$ and
- $\Delta_i^n S \Delta_{i+1}^n S < 0$

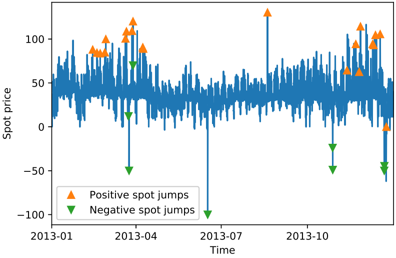
where $\hat{\sigma}^2$ is the multipower variation of order 20.

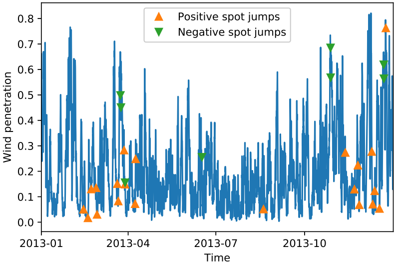
We find 84 positive jumps and 30 negative jumps.

They propose a consistent estimator for β : we find 7718.84 $year^{-1}$.

The size of the jumps are estimated with $\Delta_i^n S$ at a time of a jump and the empirical distribution is considered in our model.

Jump detection versus wind penetration





Jumps in German spot price (left) and wind penetration index (right).

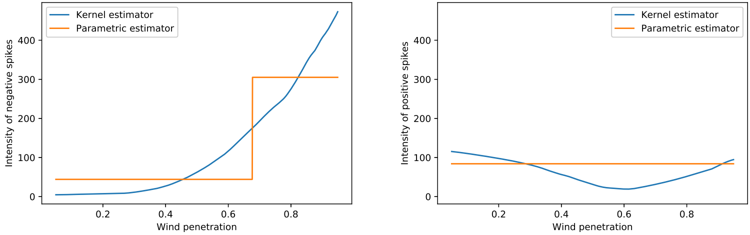
Intensity process modeling

- Positive and negative jumps are distinguished with one intensity function for each, $\lambda_t^- = q^-(WP_t)$ and $\lambda_t^+ = q^+(WP_t)$.
- Testing negative jumps intensity function as constant is rejected

$$\Longrightarrow \text{ Model} : \lambda_t^- = \lambda_{low}^- \mathbf{1}_{WP_t \leq WP_{thre}} + \lambda_{up}^- \mathbf{1}_{WP_t \geq WP_{thre}}$$

- but is not for the positive jumps intensity

$$\Longrightarrow \text{ Model} : \lambda_t^+ = \lambda^+.$$



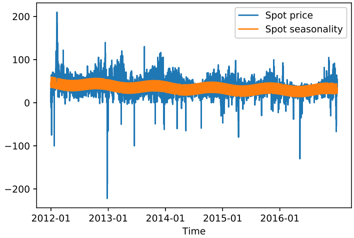
Kernel estimators for q^- (left) and q^+ (right).

Estimating the continuous part of the spot model (1/2)

$$S_t^c = \Gamma_{1,t} + X_{1,t}$$

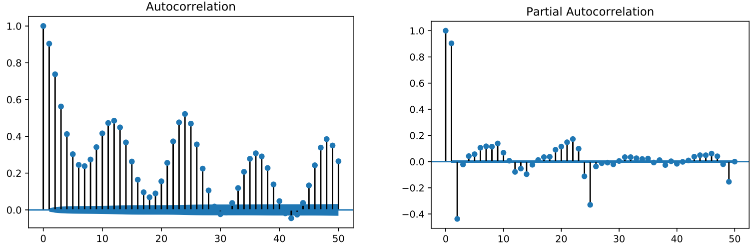
with

- $\Gamma_{1,t} = c_{0,1} + c_{1,1}t + c_{2,1}t^2 + c_{3,1}\cos\left(\frac{\tau_{0,1}+2\pi t}{365\times 24}\right) + c_{4,1}\cos\left(\frac{\tau_{1,1}+2\pi t}{7\times 24}\right) + c_{5,1}\cos\left(\frac{\tau_{2,1}+2\pi t}{24}\right)$ a seasonality function estimated after filtering the spikes using a L_2 minimization and
- X_1 a continuous autoregressive process (CAR) of order 24 estimated using the method of Benth and Benth (2012).

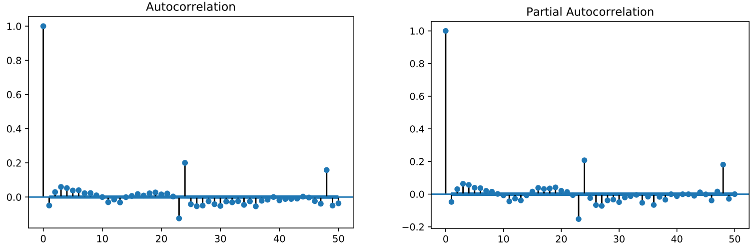


Seasonality function of the filtered spot price.

Estimating the continuous part of the spot model (2/2)



Autocorrelation and partial autocorrelation of the deseasonalised and filtered spot price.



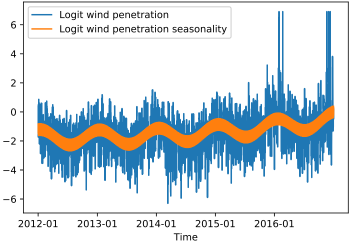
Autocorrelation and partial autocorrelation of the spot price residuals.

Estimating the wind penetration model (1/2)

$$WP_t = expit\left(\Gamma_{2,t} + X_{2,t}\right)$$

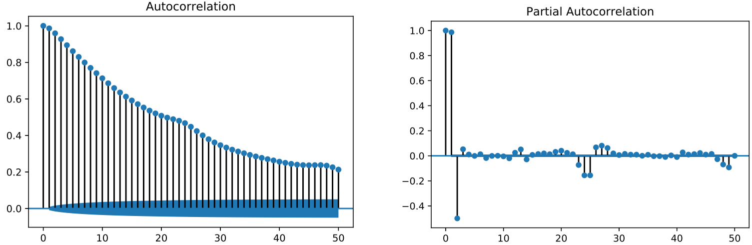
with

- Γ_2 is a seasonality function parametrized in a same way than Γ_1,
- X_2 is a CAR process of order 24 correlated to X_1.

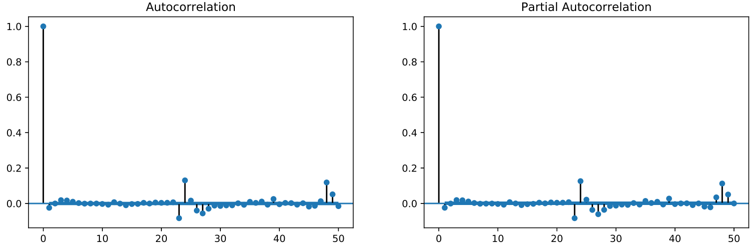


Seasonality function of the logit wind penetration.

Estimating the wind penetration model (2/2)



Autocorrelation and partial autocorrelation of the deseasonalised logit wind penetration.



Autocorrelation and partial autocorrelation of the logit wind penetration residuals.

Numerical results on the incomes of a distributor

$$Incomes = \int_0^T QWP_t C_t (S_t - K)\, dt$$

where C_t is the total load assumed to be deterministic, $Q = 1\%$ and $K = 30$ €/MWh. Two models are considered.

Model	Simple correlation	Negative spikes dependence
Expectation	$[742.5, 754.8]$	$[642.1, 654.5]$
VaR 95%	$[-906.6, -880.9]$	$[-1009.7, -978.9]$
VaR 99%	$[-1596.8, -1553.5]$	$[-1697.9, -1647.6]$
ES 95%	$[-1324.0, -1293.7]$	$[-1426.3, -1395.8]$
ES 99%	$[-1932.8, -1876.8]$	$[-2036.1, -1979.5]$

Different risk measures (k€) for the portfolio simulated between the 1^{st} of January 2017 and the 31^{st} of December 2017 10,000 times. Confidence intervals are computed with classical bootstrap methods.

Conclusion

- We proposed a model of dependence, simple enough to understand and to simulate,
- and a method to estimate it in an optimal way.
- This model is adapted for the modeling of the electricity spikes and allows to add a dependency component in the model of Deschatre et al. (2018).
- It confirms statistically that high wind penetration level has an impact on the negative spikes frequency.
- Taking into account this dependence has a strong impact on the different risk measures of a simple contract, and can not be neglected.

Thank you for your attention.

Do you have any questions ?

Bibliography I

Barlow, M. T. and Yor, M. (1982). Semi-martingale inequalities via the garsia-rodemich-rumsey lemma, and applications to local times. *Journal of functional Analysis*, 49(2) :198–229.

Benth, F. E. and Benth, J. Š. (2012). *Modeling and pricing in financial markets for weather derivatives*, volume 17. World Scientific.

Deschatre, T., Féron, O., and Hoffmann, M. (2018). Estimating fast mean-reverting jumps in electricity market models. *arXiv preprint arXiv :1803.03803*.

Houdré, C. and Reynaud-Bouret, P. (2003). Exponential inequalities, with constants, for u-statistics of order two. In *Stochastic inequalities and applications*, pages 55–69. Springer.

Lacour, C., Massart, P., and Rivoirard, V. (2016). Estimator selection : a new method with applications to kernel density estimation. *Sankhya A*, pages 1–38.

Revuz, D. and Yor, M. (2013). *Continuous martingales and Brownian motion*, volume 293. Springer Science & Business Media.

Bibliography II

Reynaud-Bouret, P. (2014). Concentration inequalities, counting processes and adaptive statistics. In *ESAIM : Proceedings*, volume 44, pages 79–98. EDP Sciences.

Yen, J.-Y. and Yor, M. (2013). Local times and excursion theory for brownian motion a tale of wiener and ito measures preface.